Attention Early Childhood STEAM
Some Trouble Spots in the Primary Grades Mathematics Curriculum

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This insightful research study by Collaborative Advisory Council member Fredricka Reisman, PhD, demonstrates important strategies to promote understanding of math concepts for primary grade students. In this article, the importance of visual observation and analysis is apparent. Visual analysis is an important STEAM thinking skill identified by the Collaborative. Dr. Reisman is Emerita Professor and Director of the Freddie Reisman Center for Translational Research in Creativity and Motivation at Drexel University. She also is founder of the Drexel School of Education and Director of the Drexel/Torrance Center for Creativity and Innovation.

As a result of task analysis, diagnostic clinical interviews with young children, and observation, the following trouble spots in teaching grades kindergarten through grade 2 mathematics have been identified. Attention is directed to misunderstandings that cause pervasive learning challenges as they neglect pedagogy that embraces the learner's perspective.

Counting Objects Versus Counting on a Number Line

Counting objects in a set leads to obtaining the cardinality of the set (the number of objects in the group). However, this does not correspond to counting on a number line (or a clock face, which is really a circular number line). When counting objects, the objects are enumerated, not the spaces between them. But on a number line or a clock face, the spaces between the numbers—represented by jumps or moves—are counted rather than the digits (numbers), dots or minute marks that designate position. This discrepancy between counting on a number line or counting objects often results in a number line count of one more than is correct. The early learner applies previous learning involving counting objects to number line counting. Thus, they start the count at the first position, which intuitively is at the 0 or on a clockface the 12 position. This discrepancy between counting on a number line and counting objects is illustrated in Figure 1.

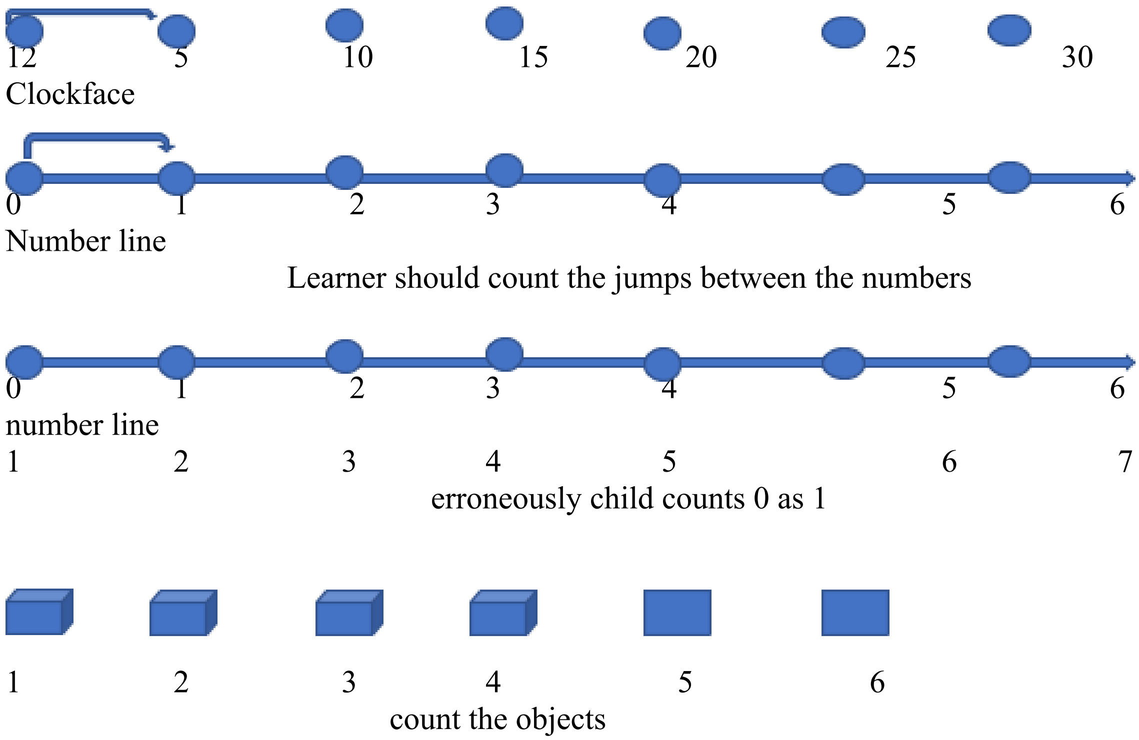

Figure 1. Discrepancy Between Number Line and Object Counting

Physical Objects and Notation

When representing the number zero concretely or with pictures, no objects are used as shown on the place value/counting board in Figure 2. However, *nothing* is represented *with something* when using digits—namely, the digit ***0***. Therefore, there is a discrepancy between the physical representation of zero and its digital representation. This discrepancy is apparent in place-value notation, when the digit zero must be used to represent none of a place. Zero is the only digit which has this discrepancy. All of the other digits, 1–9, may be represented by “something” in the real world. Students often are not intuitively aware of this discrepancy and this may cause misconceptions, especially regarding place value.

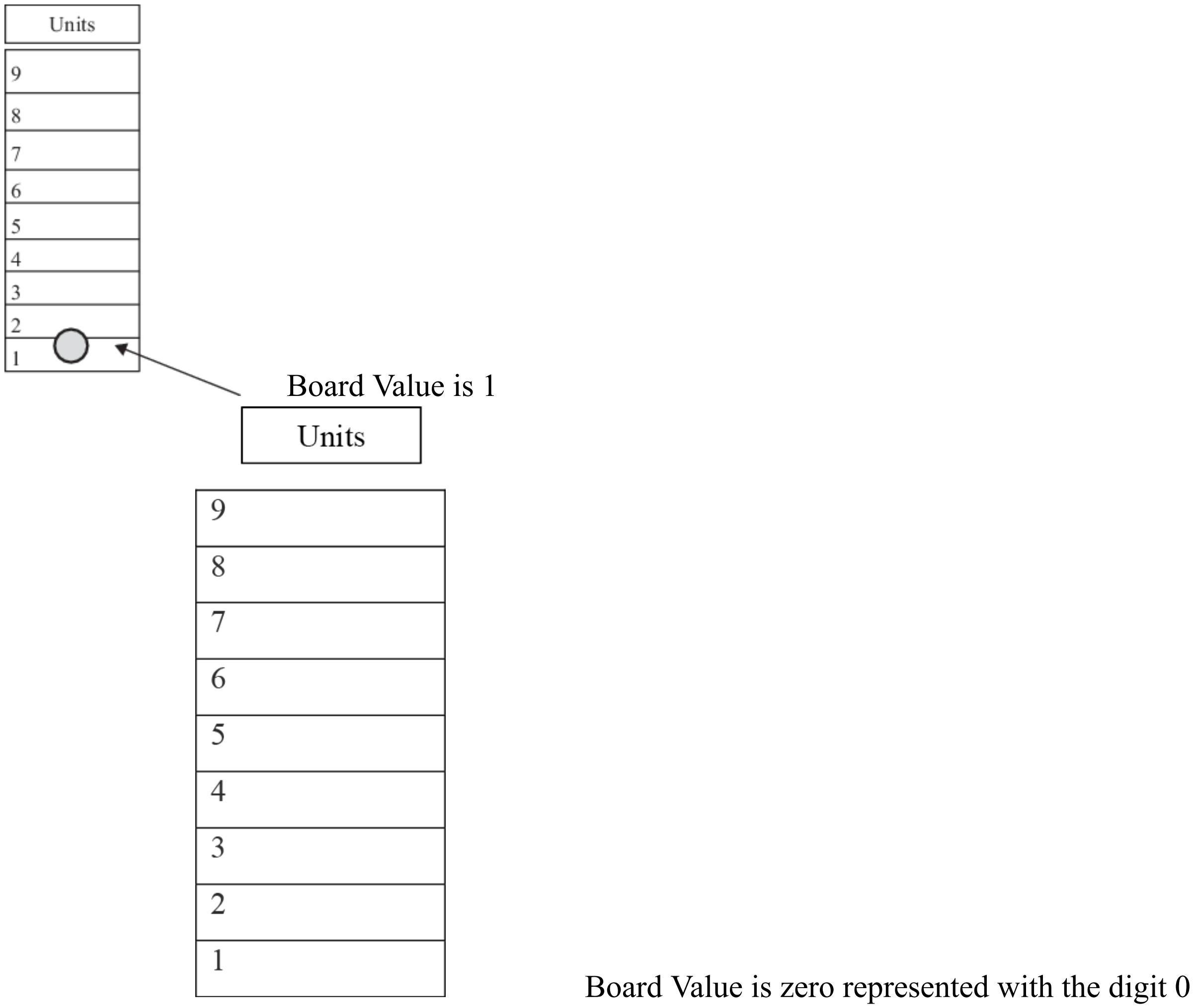

Figure 2. Discrepancy in Demonstrating Zero

Place Value/Counting Boards Versus Bundling by Ten

By the time students are in fourth through twelfth grade if they still do not understand place value, which underlies the algorithms for operations on whole numbers as well as computation with decimals, a different physical representation is needed.

The erroneous yet pervasive model of teaching place value involves bundling ten sticks that each represent “one”, which then are *exchanged* for one stick that represents “ten”. Place Value Charts, Dienes Blocks, Cuisenaire Rods, the abacus, and bead charts have been used for years. All these methods rely upon the “exchange model” that involves a change in thinking after the count of 10. In fact, we count to 9 and then make our move to the tens place; **we change place values after the count of 9 not 10.** Therefore, the bundling of tens model is not a good representation of our notational system.

In the beginning stages of learning, children do not think, “I have ten units; that’s one group of ten and no units.” Rather, initial learning takes the form of a *count-on-by-one* model and incorporates a horizontal move to the next position to the left ***after the count of nine, not ten.***

A creative approach to learning place value uses Place Value/Counting Boards, which represent a physical embodiment of the rationale for moving from the units (ones) place to the tens place to the hundreds place, etc. The Place Value/Counting Boards are a truer representation of the finite set of digits in our Hindu/Arabic numeration system (0,1,2,3,4,5,6,7,8,9).

The Place Value/Counting Boards were created by Reisman after many years of diagnosing why students had so much difficulty learning place value. Each board has nine spaces with increasing values from 1 through 9 (see Figures 3 and 4). As you make your own set of Boards out of construction paper or cardboard, it is helpful to write the digits on the board spaces with the bottom space labeled *1*, the next space labeled *2*, etc. An empty board has a value of zero. A value of one is obtained by jumping a chip onto the board to land on the *one* space. It is important to represent the board’s initial value as *zero* by starting with an empty board as shown in Figure 3.

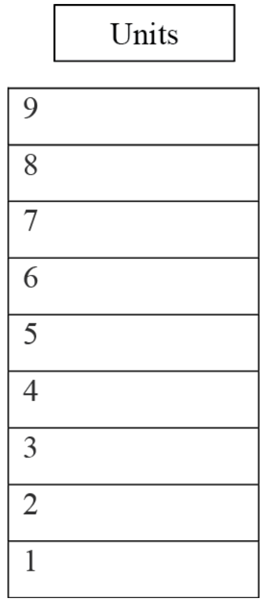

Figure 3. Board Showing Zero

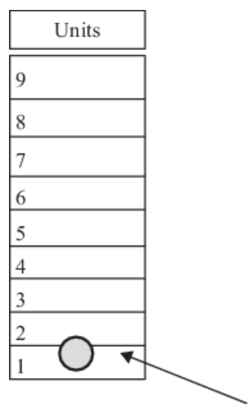

Figure 4. Board Showing Value of One

Have students practice counting board values from zero to nine by jumping a chip onto the board to count "one," moving the chip up 1 space to show "2," and so on up to "9" (see Figure 4). Continue counting one more beyond 9 (see Figure 5) and call out "ten"as the chip is moved up and off the top of the counting board. Although there is now no chip on the board, set a rule that once the count starts the counting board continues to have value, especially when the count is greater than nine and the units board appears empty. Otherwise students might focus on the visual emptiness of the board and forget that they had been counting upward beyond nine to ten, with the chip now sitting in the space just above the Board (see Figure 6).

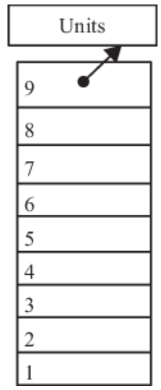

Figure 5. Counting Board Showing Impending Count of "Ten"

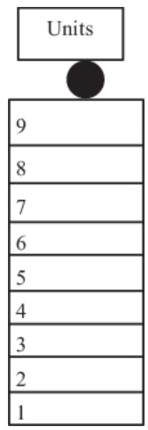

Figure 6. Counting Board Showing Count of "Ten"

After the student has had practice using one board, provide a second counting board. Position the second board ("tens board") to the upper left of the "units board" (see Figure 7) to continue the upward counting sequence beyond 9.

The board to the upper left represents the "tens" place value. Each space on the "tens" board is worth 10, just as one space on the "hundreds board" is worth 100, etc. Remind students that as the chip is moved up a space, the board value increases by its value. Thus, moving up a space on the tens board is 20, 30, 40, etc.

Present the problem-solving situation of what the count will be if the chip is moved up 1 move beyond 9; it is now just above the units board. Move the chip horizontally to the left onto the bottom space of the board to the upper left (see Figure 7). Since only a horizontal move and no upper move occurred, there was no increase in value. Consequently, the value of the bottom space on the board in the "tens" place is 10; the boards show 1 "ten" and 0 "units" or 10.

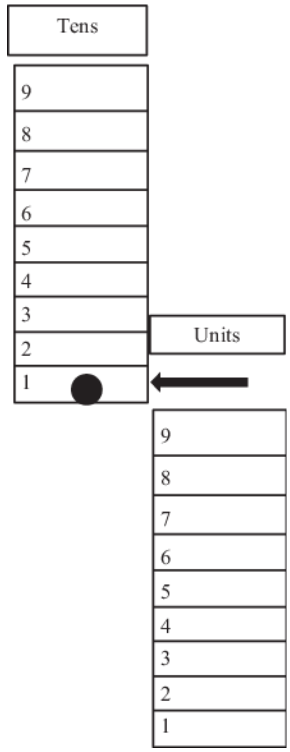

Figure 7. Units and Tens Counting Boards Showing that the Horizontal Move from Units to Tens Remains "Ten"

Offer the student a second chip and instruct them to show the number 11. He or she should move a second chip onto the 1-space of the units' board. Be sure that students understand that 11 should be shown with a chip on the 10-space of the "tens" board and a chip on the 1-space of the Units board. If a student attempts to show 11 by moving the second chip to the 20-space on the "Tens" board, use the following diagnostic questions to help them understand that they are showing 21, not 11. Ask: if you move the chip on the tens board up a space, what will the value of the boards be? What number would you write under the "Tens" board if your chip is in the second space? Instruct students to show 11 using both boards (Figure 8)

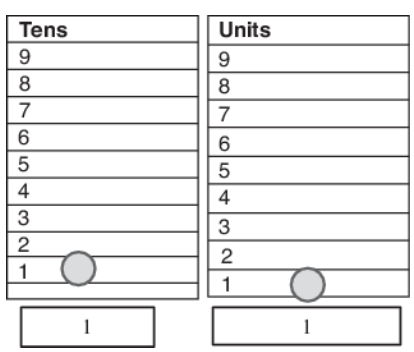

Figure 8. Tens and Units Boards Representing "Eleven"

Repeat the process as students count to 19. At the count of 20, the counting boards show a chip in the ones space on the “Tens” board with a second chip sitting atop the “Units” board. Make sure that students understand that just as two digits cannot be written in one space, two chips (representing digits) cannot be in one space. The chip with a count of 20, sitting above the units board needs to move horizontally to the tens board, maintaining the value of 20, which results in two chips in the bottom tens board space. This cannot be, just as we do not write two digits, one over the other, in the same number position. So how to solve the dilemma?

There now is the opportunity to allow a 2-for-1 exchange, by exchanging the two chips worth 10 each for one chip placed on the second space whose value is 20. Thus, in this case a 2 for 1 exchange is appropriate.

Place-Value Placement in the Instructional Sequence

The numbers resulting from *ten* ones (10 x 1), one *ten* (1 x 10), *ten* tens (10 *x* 10), *ten hundreds* (10 x 100), and so on are products. A product is the result of performing the binary operation of multiplication on two numbers; the product is the answer to a multiplication problem. Place value is a product. Yet, when is place value taught? And when is multiplication taught? Place value is typically a first-grade topic, while multiplication is introduced at the end of second grade and most usually in grade three. Unless the prerequisite of multiplication is taught either prior to or simultaneously with place value, only a rote learning of the place-value names occurs without a meaningful interpretation of how the names of the place values are generated.

Teach Time to the Minute First

In the typical time telling sequence counting by fives such as “five minutes after the hour” or “a quarter after the hour” or “half after” come first. But these topics involve multiplication and fractions that normally are introduced in 2nd and 3rd grades. Children learn to count by ones first not last, and they need to count to 60 to fully understand minutes on a clockface. Thus, time telling instruction should focus on children learning to tell time to the precision of a minute in grade one rather than the end of grade two or in grade three where counting by fives and fractions should occur.

Discrepancy Between Place Values and Number Line Values

Place values get larger to the left while number line values get larger in a right direction (Figure 9).

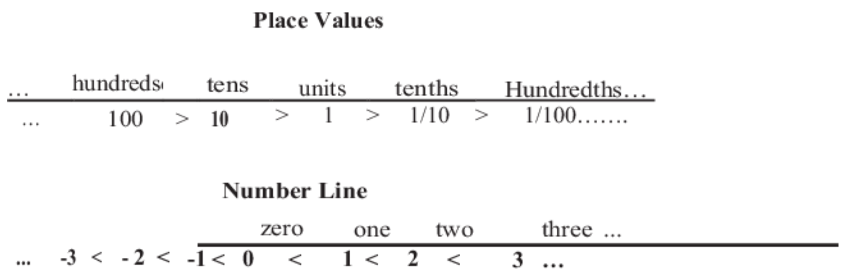

Figure 9. Place Value Notation and Use of a Number Line

Perhaps using a vertical number line would help alleviate some of the confusion. Children have early experiences with vertical number lines—thermometers for showing temperature and graphs for recording measurement. Using the y-axis instead of the x-axis would allow the same manipulations (such as counting, computing, and sequencing) but would not interfere with learning place-value notation. Also confusing is the function of the units place value as the fulcrum of our decimal notation system, in comparison with zero as the fulcrum for negative and positive integers.

Summary

These components of the mathematics curriculum that are typical for primary grade children are learning trouble spots that are not usually apparent to the instructor. This is because instruction does not consider the young child's perspective but is enmeshed in what the adult assumes the child knows. It is hoped that this disclosure uncovers these trouble spots so they may be overtly addressed in instruction.

Article content adapted from:

Reisman, F. & Severino, L. (2021). *Using Creativity to Address Dyslexia, Dysgraphia, and Dyscalculia:Assessment and Techniques.* UK: Routledge, ***Taylor & Francis Group***

Reisman, F.K. and Kauffman, S.H. (1980). Mathematics Instruction for Children with Special Needs. Columbus, Ohio: Charles E. Merrill.

Reisman, F.K. Children's errors in telling time and a recommended teaching sequence. The Arithmetic Teacher, MARCH 1971, Vol. 18, No. 3 (MARCH 1971), pp. 152-155

An Evaluative Study of the Effects of Cognitive Acceleration in Mathematics in the Early School Years. 68 67p.; Based on Ph.D. Dissertation, Syracuse University